

# Focussing in on $H_0$

(Dedicated to Allan Sandage for his seventieth birthday)

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Six independent methods yield a Virgo cluster modulus of  $(m - M) = 31.66 \pm 0.08$ . This inserted into the Hubble diagram of clusters, whose relative distances to the Virgo cluster are known, gives  $H_0 = 54 \pm 4$ . The result is independent of any observed or inferred velocity of the Virgo cluster and holds out to  $\sim 10\,000 \text{ km s}^{-1}$ . An analogous use of the Fornax cluster is not yet possible because of its unknown structure in space and its poorly understood distance relative to other clusters. Field galaxies, corrected for Malmquist bias, give  $H_0 = 53 \pm 3$ . The results of purely physical distance determinations cluster around  $H_0 = 55 \pm 10$ . These results may be compared with the high-weight determination of  $H_0 = 58.5 \pm 3$  from Cepheid-calibrated SNeIa and their Hubble diagram out to  $\sim 30\,000 \text{ km s}^{-1}$  (Saha 1996). The agreement of the independent methods suggests that the external error of an adopted value of  $H_0 = 58$  is equal or less than 13%.

## 1. Introduction

The determination of distances of individual galaxies and the calibration of the Hubble constant  $H_0$  are two connected, but fundamentally different problems. The calibration of  $H_0$  requires in addition to reliable galaxy distances the demonstration that the galaxies under consideration do partake of the expansion of the Universe and that their velocities are not dominated by, for instance, peculiar or virial motions. Moreover, the question must be answered whether the expansion can be characterized by a single value of  $H_0$ , or whether  $H_0$  is scale-variant or even a stochastic variable.

The ensuing strategy is clear. One must construct a Hubble diagram plotting for suitable objects (e.g. SNeIa, galaxies, or clusters)  $\log cz$  versus  $\log$  (relative distance). The relative distance is approximated by the apparent magnitude of “standard candles”. (Standard candles are to be objects whose scatter in absolute magnitude is  $< 0.3$  mag to avoid gross effects of Malmquist bias.) Alternatively, relative distances of non-standard candles, for instance clusters, can come from various distance indicators, whose zero-point calibration is not needed. In either case the resulting Hubble diagram provides two basic informations. (1) Do the objects under consideration partake of the *linear* expansion (corresponding to a ridge line slope of 0.2!), and (2) what is the magnitude scatter about the ridge line? This scatter is decisive for the precision with which the intercept of the ridge line – and hence  $H_0$  – can be determined.

Once the Hubble diagram is found satisfactory out to a maximum recession velocity, the next step is to determine the absolute magnitude or the linear distance of one or more objects defining this Hubble diagram. The resulting value of  $H_0$  is valid then out to the same maximum recession velocity.

The most successful application of this strategy to date comes from the Hubble diagram of SNeIa, which is defined out to  $30\,000 \text{ km s}^{-1}$  – and hence provides a truly cosmic value of  $H_0$  – and its calibration by means of seven SNeIa with known Cepheid distances (Saha 1996).

An independent application of essentially the same strategy is discussed in Section 2. Here a Hubble diagram is constructed from 31 clusters whose distances are known relative to the Virgo cluster. By inserting the best available distance of the latter one can

transform the Hubble diagram into a diagram of recession velocity versus linear distance, allowing to read off  $H_0$  at any distance up to 11 000 km s $^{-1}$ .

In Section 3 the still poorly determined distance of the Fornax cluster as well as the evidence from the Coma cluster are discussed. The evidence of  $H_0$  from field galaxies and physical distance determinations is compiled in Sections 4 and 5. The conclusions follow in Section 6.

## 2. The Global Value of $H_0$ from the Virgo Cluster Distance

### 2.1. The Hubble Diagram of Clusters out to 11 000 km s $^{-1}$

Cluster distances relative to the Virgo cluster are available for 17 clusters from various methods like the Tully-Fisher (TF) method, the  $D_n - \sigma$  relation, and first-ranked cluster galaxies (for a compilation see Jerjen & Tammann 1993). In addition, carefully determined relative TF distances are available for 24 clusters from Giovanelli (1996a). The latter list does not include the Virgo cluster, but since eight clusters are in common, the two lists can be combined with a mean error of only 0.05 mag. The resulting list of 31 cluster distances relative to the Virgo cluster is given in Table 1. The double cluster A 2634/66 is not used here because Giovanelli only gives a distance for A 2634.

The Hubble diagram of the 31 clusters is shown in Fig. 1. Clusters with  $v_0 < 3000$  km s $^{-1}$  are corrected for a Virgocentric infall model with a local infall velocity of 220 km s $^{-1}$ . More distant clusters do not partake of the local motion with respect to the CMB. They are therefore corrected for a CMB vector of 630 km s $^{-1}$ . The above dividing limit of 3000 km s $^{-1}$  is an educated guess (cf. Jerjen & Tammann 1993, Giovanelli 1996a); the exact choice has no effect on the following conclusions.

The ridge line in Fig. 1 is represented by

$$\log v^{\text{CMB}} = 0.2 \left[ (m - M) - (m - M)_{\text{Virgo}} \right] + 3.068 \pm 0.024, \quad \sigma = 0.13. \quad (2.1)$$

The slope of 0.2 is forced. A free fit gives a slope of  $0.205 \pm 0.005$  (or  $4.82 \pm 0.20$  for the inverse regression) in statistically perfect agreement with a linear expansion.

Simple transformation of equation (1) gives

$$\log H_0 = \log v^{\text{CMB}} - \log r_{\text{Mpc}} = -0.2(m - M)_{\text{Virgo}} + (8.068 \pm 0.024) \quad (2.2)$$

Inserting the distance modulus of the Virgo cluster into equation (2.2) immediately yields the value of  $H_0$  out to 11 000 km s $^{-1}$ . The next section is therefore devoted to a discussion of the Virgo cluster distance.

If the strongly deviating Eridanus cluster (cf. Fig. 1) is excluded the constant term in eq. (2.1) changes only marginally to be  $3.070 \pm 0.020$  and the scatter reduces to  $\sigma = 0.11$  mag. This scatter restricts the peculiar motions of the cluster centers quite severely. Even an optimistic error of the relative distances of 0.08 mag leaves a scatter of only 0.07 mag due to peculiar motions, which translates into a mean (one-dimensional) peculiar motion of  $\sim 350$  km s $^{-1}$  (cf. Jerjen & Tammann 1993).

It is noted in passing that eq. (2.1) also predicts the Virgo cluster velocity in the CMB frame at a relative distance of 0, i.e.  $1169 \pm 30$  km s $^{-1}$ . This is the velocity one would observe in the absence of all local peculiar or streaming velocities. Comparing this value with the actually observed cluster velocity of  $v_0 = 1050 \pm 35$  km s $^{-1}$  (Binggeli, Popescu & Tammann 1993) or correspondingly  $v_{\text{LG}} = 922 \pm 35$  km s $^{-1}$ , one obtains a Virgocentric infall velocity of the Local Group of  $v_{\text{infall}} = 247 \pm 46$  km s $^{-1}$  (cf. Jerjen & Tammann 1993). We take this as a confirmation of  $v_{\text{infall}} = 220$  km s $^{-1}$  adopted above (Tammann and Sandage 1985). The result is also compatible with  $v_{\text{infall}} = 275 \pm 90$  km s $^{-1}$  from the Hubble diagram of SNeIa (Hamuy et al. 1996).

TABLE 1. Mean cluster distances relative to the Virgo cluster

cluster	$v_{220}$	$v^{\text{CMB}}$	$(m - M) - (m - M)_{\text{Virgo}}$	source*
Ursa Major	1270		0.37	JT, G
Fornax	1338		0.23	JT, G
Eridanus	1522		0.93	G
MDL 59	2636		1.85	G
Antlia	2767		1.93	G
N 3557	2973		2.08	G
ESO 508	3029		1.83	G
Centaurus	3043		2.18	JT, G
Pegasus	3517		2.46	JT, G
Hydra I	4050		2.73	JT, G
Pavo	4055		2.60	G
Pavo II	4444		2.78	G
A 262	4664		2.92	G
Pisces	4717		2.99	JT
N 507	4808		2.97	G
A 3574	4817		3.03	G
N 383	4865		3.04	G
Cancer	5026		3.07	JT, G
Perseus	5178		3.23	JT
Zw74-23	6308		3.60	JT
A 1367	6777		3.72	JT, G
A 400	6920		3.95	JT, G
A 1656	7185		3.84	G
Coma	7188		3.80	JT
A 539	8630		4.39	JT
A 2634/66	8265		4.16	JT, G
A 2199	8996		4.45	G
A 2197	9162		4.45	G
A 1185	10808		5.03	JT
A 2147	10981		4.98	JT
Hercules	11058		4.71	JT

\*sources: JT=Jerjen & Tammann (1993); G=Giovanelli (1996a)

*It must be stressed that the determination of  $H_0$  from eq. (2.2) depends on the quality of the Virgo cluster distance, but it is totally independent of any observed or inferred velocity of that cluster.*

## 2.2. The Virgo Cluster Distance from Cepheids

Observations of Cepheids in Virgo cluster galaxies require the Hubble Space Telescope (HST) because due to resolution problems essentially all stars selected from the ground as “single” turn out to be blends or multiples on HST images.

The first HST Cepheid distance of a galaxy in the Virgo complex was for NGC 4321 (M 100; Freedman et al. 1994; see also Ferrarese et al. 1996). The distance was surprisingly small, i.e.  $17.1 \pm 1.7$  Mpc, and was precipitately interpreted as the distance of the Virgo cluster (Mould et al. 1995; Kennicutt, Freedman, & Mould 1995), although already de Vaucouleurs (1982) had derived an exceptionally small (relative) distance of this galaxy, and in spite of the wide angular separation of the Virgo spiral members and the correspondingly important depth effect. The next two spirals in the Virgo complex with HST Cepheid distances did seemingly confirm the small cluster distance (Saha et

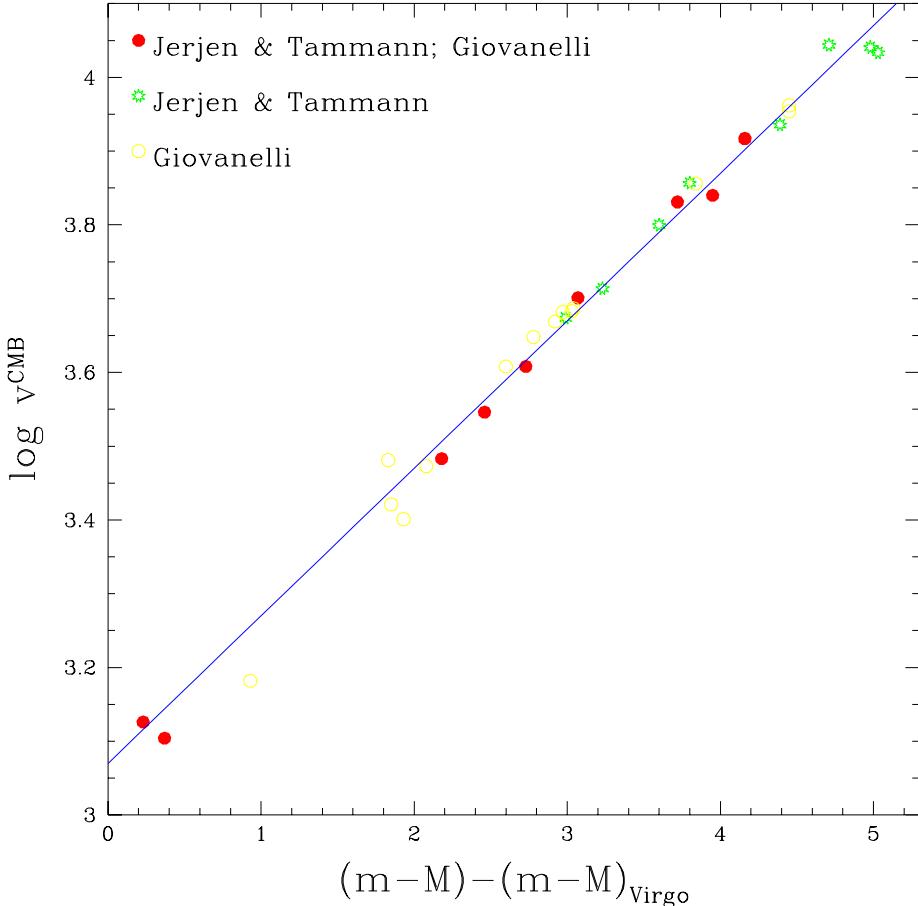


FIGURE 1. Hubble diagram of 31 clusters with known relative distances. The data were taken from Jerjen & Tammann (1993; asterisks) and Giovanelli (1996a, open circles). Nine clusters are listed in both sources (filled circles). The abscissa gives the distance modulus relative to the Virgo cluster and the ordinate the log of the recession velocity referred to the CMB. For “local” clusters with  $v_0 < 3000 \text{ km s}^{-1}$  the velocities are referred to the centroid of the Local Group and corrected for Virgocentric infall.

al. 1996a, 1996b). Yet all galaxies were selected to be “easy”, i.e. that their resolution had to be at least “good to fair” (Sandage & Bedke 1988), and there was therefore an a priori expectation that they were nearer than average. (See also the evidence from SN Ia models, Branch, Nugent, & Fisher 1996.)

Indeed a fourth Virgo cluster spiral, NGC 4639, which was selected for observations with HST because it had produced a supernova of type Ia, and which is an unquestionable cluster member on the basis of its low redshift, gives a much larger Cepheid distance, i.e.  $25.1 \pm 2.5 \text{ Mpc}$  (Sandage et al. 1996). The four Virgo galaxies with Cepheid distances are compiled in Table 2. Two of the galaxies lie in subcluster A; the remaining ones in the southeastern W cloud. The latter are therefore quite uncertain cluster members.

It is clear that the four distances in Table 2 cannot be averaged. All one can conclude is that the cluster center lies somewhere between 17 and 25 Mpc. At least a dozen randomly selected cluster members would be needed to define a reliable Cepheid distance of the

TABLE 2. Virgo cluster galaxies with HST Cepheid distances

	$v_0$ km s <sup>-1</sup>	subcluster	selection	$(m - M)^0$	source
NGC 4321	1464	A	resolution	$31.04 \pm 0.17$	Ferrarese et al. 1996
NGC 4496A	1568	(W)	resolution	$31.13 \pm 0.10$	Saha et al. 1996b
NGC 4536	1646	(W)	resolution	$31.05 \pm 0.15$	Saha et al. 1996a
NGC 4639	860	A	well obs. SN	$32.00 \pm 0.23$	Sandage et al. 1995

TABLE 3. Relative distance moduli between the Leo group and the Virgo cluster

Method	$\Delta(m - M)_{\text{Virgo-Leo}}$	Source
Tully-Fischer	$1.35 \pm 0.20$	Federspiel et al. (1996)
Globular clusters	$1.47 \pm 0.42$	Harris (1990)
$D_n - \sigma$	$0.97 \pm 0.29$	Faber et al. (1989)
Planetary nebulae	$1.15 \pm 0.30$	Bottinelli et al. (1991)
Velocities	$1.30 \pm 0.30$	Kraan-Korteweg (1986)
mean:	$1.25 \pm 0.13$	

cluster as such. All what one can say with certainty is that  $(m - M)_{\text{Virgo}} > 31.1$  and hence, from eq. (2.2),  $H_0 < 71 \text{ km s}^{-1} \text{ Mpc}^{-1}$ !

An auxiliary route to come to a Cepheid-based Virgo cluster distance at present is to use the Leo group which, judging from the Cepheids in NGC 3368 (M96; Tanvir et al. 1995) and NGC 3351 (M95; Graham et al. 1996), lies at a mean distance of  $(m - M)_{\text{Leo}} = 30.22 \pm 0.12$  (increased by 0.05 mag for a zeropoint offset of HST photometry of relatively bright stars following Saha et al. 1996). The relative distance between the Leo group and the Virgo cluster can be estimated using different distance indicators (Table 3).

With the distance of the Leo group and the modulus difference from Table 3 the distance modulus of the Virgo cluster becomes

$$(m - M)_{\text{Virgo}}^0 = 31.47 \pm 0.21. \quad (2.3)$$

For brevity we will refer to this value in the following as the “Cepheid distance of the Virgo cluster”; this is additionally supported by the fact that the value lies midway between the directly determined Cepheid distances of the Virgo cluster members in Table 2.

A first shot at  $H_0$  is obtained by combining eqs. (2.2) and (2.3), leading to  $H_0 = 60 \pm 10 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (external error).

### 2.3. The Virgo Cluster Distance from 21 cm Line Widths

The 21 cm line width-absolute magnitude relation (or Tully-Fisher [TF] relation) has been applied many times to derive a Virgo cluster distance with variable success. The principal difficulty is that a *complete* sample of cluster spirals is required to avoid the well known “Teerikorpi cluster bias” (Teerikorpi 1987, 1990), which systematically leads to an underestimate of the cluster distance if only the brightest cluster members are used. Two previous studies using complete samples selected from the Virgo Cluster Catalogue (VCC; Binggeli, Sandage, & Tammann 1985), gave a rather large distance

in nearly perfect mutual agreement, if the different adopted zero-point calibrations of the TF relation are adjusted (Kraan-Korteweg, Cameron, & Tammann 1988; Fouqué et al. 1990).

It has been suggested to use the inverse TF relation (i.e. line widths  $w$  versus absolute magnitude, the latter as independent variable to guard against selection bias of magnitude-selected samples, which the complete Virgo sample is not). While the direct method is affected by magnitude errors, the indirect method is sensitive to errors of the line widths. In the Virgo sample the magnitudes are quite good, and it seems that the line widths errors are more important; this speaks here against the indirect method. The most severe disadvantage of the indirect method as applied to clusters is, however, that all members must be assumed to lie at the same distance and any depth effect, which is so severe for the Virgo spirals, must be denied. This justifies in the following that only the direct method is used (see also Giovanelli 1996b).

The method requires the galaxian magnitudes to be corrected for the inclination-dependent internal absorption, which is notoriously difficult. On the assumption that the internal absorption is significantly smaller in the infrared it has been proposed to use  $I$  or  $H$  magnitudes for the TF relation (e.g. Aaronson et al. 1982). However, for an ensemble of stars embedded in dust the absorption decreases with wavelength much slower than with an  $1/\lambda$ -law. A systematic investigation of a complete sample of 171 Virgo galaxies and 46 Fornax galaxies in  $UBVRI$  has shown that the accuracy of the TF distances of the two clusters is nearly the same for all wavelengths (Schröder 1995). The situation may be somewhat more favourable for  $H$  magnitudes, but they are available only for a small fraction of the Virgo spirals.

The 171 Virgo and 46 Fornax spirals offer the possibility to study the realistic wavelength dependence of the internal absorption by requiring that the TF relation has a minimum scatter in each wavelength and that the distances of the individual cluster galaxies are independent on average of the inclination. The result is that the internal absorptions in  $U$ ,  $V$ ,  $R$ , and  $I$  are 107, 91, 86 and 80%, respectively, of the absorption in  $B$  (Schröder 1995). The value for  $A_I$  of 80% is independently confirmed by Giovanelli et al. (1994).

The calibration of the TF relation has been dramatically improved by the advent of Cepheid distances from HST. There are now 14 Cepheid distances available of spirals suitable for the TF calibration. Two late-type *bona fide* companions of M101 can be added; the data are given in Table 4. The resulting calibration in  $B$  is shown in Fig. 2. For further details see Federspiel, Tammann & Sandage (1996).

A free linear regression to the calibrators yields a slope  $M_B \propto -6.11 \log w$ ; but a better determined slope comes from the complete sample of 49 inclined ( $i > 45^\circ$ ) spiral members of the Virgo cluster [cf. eq. (2.5)]. With this slope the weighted regression becomes

$$M_B = -6.60 \log w - (3.38 \pm 0.10) \quad (2.4)$$

with a scatter of only  $\sigma_M = 0.38$ . [A larger sample of field galaxies shows that the scatter actually depends strongly on the line width  $w$  (Federspiel, Sandage, & Tammann 1994).]

A fit of 49 Virgo cluster spirals gives

$$m_B = -6.60 \log w + (28.30 \pm 0.08), \quad \sigma = 0.57. \quad (2.5)$$

The larger scatter here is obviously due to the depth effect of the Virgo cluster.

A combination of eqs. (4) and (5) immediately gives the Virgo cluster modulus of  $(m - M)_{\text{Virgo}} = 31.68 \pm 0.13$ . However, this value depends unexpectedly strongly on the choice of the input parameters. So far the values of  $m_B$ , internal absorption  $A^i$ , and Galactic absorption  $A^0$  are taken from the RC3 (de Vaucouleurs et al. 1991;  $A^0$  actually

TABLE 4. Galaxies with Cepheid distances for the calibration of the Tully-Fisher relation

Name (1)	Hubble- type (2)	$(m - M)$ mag (3)	Source (4)	$M_B$ mag (5)	$i_{RC3}$ ° (6)	$\log w$ (7)
N224 (M31)	3	24.44	Madore & Freedman (1991)	-21.08	78	2.728
N300	5	26.67	Madore & Freedman (1991)	-18.18	44	2.296
N598 (M33)	5	24.63	Madore & Freedman (1991)	-18.90	55	2.357
N925	5	29.84*	Silbermann et al. (1996a)	-19.80	58	2.382
N1365	4	31.30*	Silbermann et al. (1996b)	-21.36	59	2.648
N2403	5	27.51	Tammann & Sandage (1968)	-19.13	62	2.415
			Madore & Freedman (1991)			
N3031 (M81)	3	27.80*	Freedman & Madore (1994)	-20.46	65	2.667
N3351 (M95)	3	30.01*	Graham et al. (1996)	-19.77	50	2.538
N3368 (M96)	2	30.32*	Tanvir et al. (1995)	-20.47	50	2.649
N3621	5	29.85*	Macri et al. (1996)	-19.55	57	2.509
N4321 (M100)	5	31.04*	Ferrarese et al. (1996)	-21.16	36†	2.725
N4536	4	31.11*	Saha et al. (1996a)	-20.49	66	2.548
N4639	3	32.00*	Sandage et al. (1996)	-20.05	50	2.617
N5204	7	29.34	like M101	-17.90	57	2.131
N5457 (M101)	5	29.34*	Sandage & Tammann (1974)	-21.03	27†	2.665
			Kelson et al. (1995)			
N5585	7	29.34	like M101	-18.39	52	2.260

\* Cepheid distance from HST

† the inclination is from a velocity map of the galaxy

TABLE 5. The dependence of the Virgo TF modulus on input parameters

solution	input parameters	$(m - M)_{\text{Virgo}}$
1	$m_B$ , $A^i$ , $A^0$ from RC3, $\log w$ from LEDA	$31.68 \pm 0.13$
2	$m_B$ from Schröder 1995, others like solution 1	$31.71 \pm 0.13$
3	like solution 1, but $\log w$ from Huchtmeier & Richter (1989)	$31.70 \pm 0.14$
4	like solution 1, but $A^i$ from RSA	$31.86 \pm 0.13$
5	like solution 4, but $\log w$ from Huchtmeier & Richter (1989)	$31.88 \pm 0.13$
6	like solution 1, but $A^0 = 0$	$31.73 \pm 0.13$
mean 1-5		$31.77 \pm 0.13$

from Burstein & Heiles 1984), and the line widths  $w$  from LEDA (Lyon Extragalactic Data Base). If these input parameters are taken from other sources, one obtains distance moduli as given in Table 5.

The result in Table 5 depends on the *assumption* that field and cluster galaxies obey the same TF relation. This assumption can now be tested with the complete *UBVRI* photometry of Schröder (1995). It turns out that the cluster galaxies are redder in  $(B - I)$  (corrected for internal and galactic reddening) at a given line width  $w$  than the 16 calibrators which are predominantly field galaxies; the former are also H I-deficient. The hydrogen deficiency  $D_{\text{HI}}$  is here defined following Yasuda, Fukugita, & Okamura

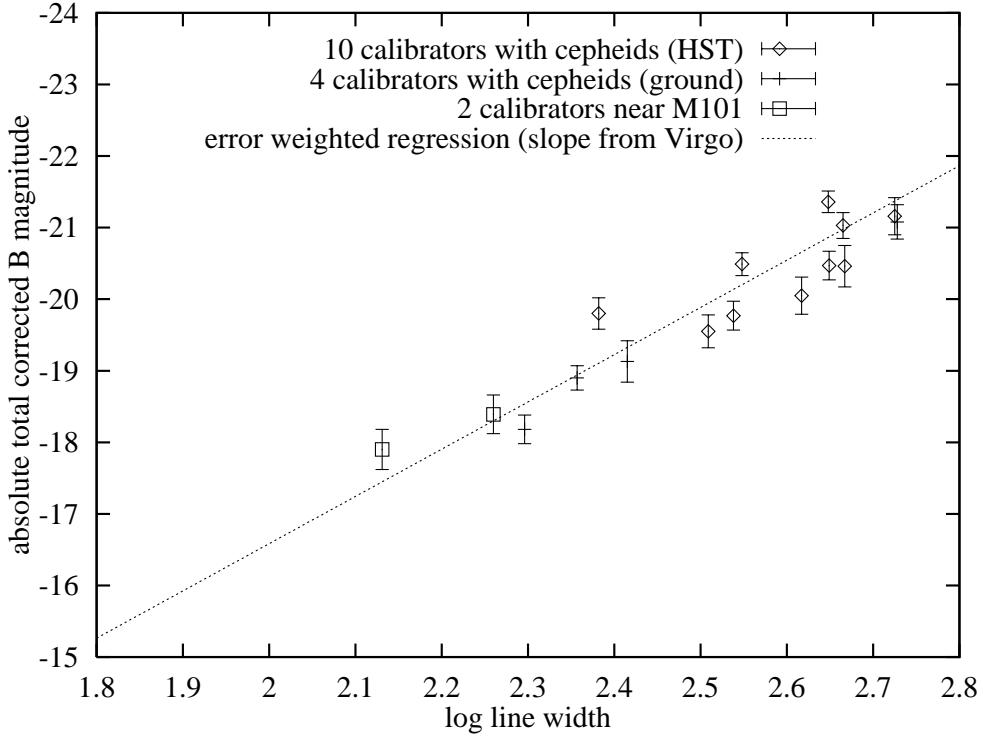


FIGURE 2. Tully-Fisher relation for the calibrators. Ten calibrators have Cepheid distances determined with HST, 4 have ground-based Cepheid distances, 2 are members of the relatively tight M 101 group without individual Cepheid distances. The error bars show the total errors in absolute magnitude which were used as weights for the regression. The dotted line represents the adopted calibration of eq. (2.4)

(1996). The colour residuals  $\Delta(B - I) = (B - I)_{\text{obs.}} - (B - I)_{\text{calib.}}$  at fixed line width correlate well with  $D_{\text{HI}}$  (Schröder & Tammann 1996).

The consequence for the individual TF distances of the Virgo galaxies is clear. In  $U$ ,  $B$ , and  $V$  cluster galaxies with small  $\Delta(B - I)$  give relatively small distances, while they give somewhat large distances in  $I$ . For red galaxies [large  $\Delta(B - I)$ ] the opposite holds. The individual distances in  $R$  show the minimum dependence on  $\Delta(B - I)$ . Analogously HI-normal galaxies give smaller distances than HI-deficient ones in  $U$ , whereas the opposite is true in  $V$ ,  $R$ , and  $I$ . Here the smallest  $D_{\text{HI}}$  dependence is found in the  $B$ -band. The net effect is that the *mean* cluster distance decreases seemingly with increasing wavelength, the  $I$  modulus being 0.5 mag smaller than the  $U$  modulus (cf. column 2 of Table 6; Schröder & Tammann 1996).

From this it is clear that the systematic difference of the calibrators and the cluster members in colour ( $B - I$ ) and in HI-deficiency requires the cluster galaxies to be reduced to the mean colour ( $B - I$ ) of the calibrators and to  $D_{\text{HI}} = 0$ . The resulting distance moduli are shown in Table 6.

For the internal absorption it has been assumed here that  $A_{\lambda}^i = f_{\lambda} A_{\text{B}}^i$  with  $f_{\lambda} = 1.07$ , 1.00, 0.91, 0.86, and 0.80 for  $U$ ,  $B$ ,  $V$ ,  $R$ , and  $I$ , respectively (cf. above). The moduli were calculated with five different sets of input parameters, corresponding to the five solutions in Table 5. Only the *mean* values of the five solutions are shown in Table 6. [The moduli in Schröder & Tammann (1996) were smaller by 0.15 mag because the values of  $A^i$  were 15% higher (resulting in a decrease of 0.05 mag) and because only the modulus

TABLE 6. Mean corrected TF Virgo moduli as a function of wavelength

passband	$(m - M)_{\text{Virgo}}$ uncorrected	$(m - M)_{\text{Virgo}}$ corrected for $\Delta(B - I)$	$(m - M)_{\text{Virgo}}$ corrected for $D_{\text{HI}}$
(1)	(2)	(3)	(4)
U	31.88	31.66	31.82
B	31.76	31.62	31.76
V	31.70	31.63	31.74
R	31.66	31.63	31.72
I	31.60	31.63	31.69
mean	31.72	31.63	31.75

corresponding to solution 1 was shown.] As can be seen in Table 6 the homogenization by colour of cluster galaxies and calibrators yields consistent cluster moduli, whereas the homogenization by  $D_{\text{HI}}$  leaves a residual drift with wavelength of 0.13 mag.

The best one can do is to average the mean corrected moduli columns (3) and (4) in Table 6 to obtain

$$(m - M)_{\text{Virgo}} = 31.69 \pm 0.15, \quad (2.6)$$

where the error is the estimated internal error.

The general conclusion is that the application of the TF relation is considerably more intricate than often realized. It not only takes multicolour information for *complete* cluster samples, but the result is also sensitive to the input parameters. The Virgo modulus in  $B$  is too large by 0.07 mag, the one in  $I$  too short by 0.09 mag. These values may change from cluster to cluster depending on the colour excess and the H I-deficiency of the spiral members.

#### 2.4. The Virgo Cluster Distance from SNe Ia

Hamuy et al. (1996) have determined the mean apparent peak magnitude of five blue, particularly well observed SNe Ia in the Virgo cluster, obtaining  $m_B(\text{max}) = 12.16 \pm 0.20$  mag and  $m_V(\text{max}) = 12.07 \pm 0.20$  mag. In perfect agreement a somewhat larger sample of eight SNe Ia, including objects with older photometry, gives  $m_B(\text{max}) = 12.10 \pm 0.13$  mag and  $m_V(\text{max}) = 12.11 \pm 0.16$  mag (Sandage & Tammann 1995). Taking the larger sample, because it is less sensitive to depth effects of the Virgo cluster, and combining it with the absolute calibration, through Cepheids observed with HST, of  $M_B(\text{max}) = -19.53 \pm 0.07$  mag and  $M_V(\text{max}) = -19.49 \pm 0.07$  mag (Saha 1996), one obtains an average cluster modulus of

$$(m - M)_{\text{Virgo}} = 31.62 \pm 0.16 \text{ mag.} \quad (2.7)$$

The Cepheid-calibrated SNe Ia luminosities are in nearly perfect agreement with the results of SN models. We will briefly return to the latter in Section 5.

#### 2.5. Other Distances to the Virgo Cluster

The peak of the luminosity function (LF) of *globular clusters* (GC) has frequently been used as a standard candle. A modern calibration of the GCs in the Galaxy and in M31 combined with a compilation of published GCLFs in five Virgo ellipticals has led to a Virgo modulus of  $(m - M)_{\text{Virgo}} = 31.75 \pm 0.11$  (Sandage & Tammann 1995). Meanwhile Whitmore et al. (1995) found a very bright peak magnitude in  $V$  and  $I$  for NGC 4486,

which is well determined with HST and which corresponds, with our precepts, to a modulus of  $31.41 \pm 0.28$  (Sandage & Tammann 1996). However, the GCs in NGC 4486 have a bimodal colour distribution which is suggestive of age differences and possible merger effects (Fritze-von Alvensleben 1995; Elson & Santiago 1996). Turning a blind eye to this problem and averaging over all available GCLFs in Virgo we obtain  $(m - M)_{\text{Virgo}} = 31.67 \pm 0.15$ . We are aware that the method may still face considerable problems.

Yet there comes some reassurance from a reversal of the argument. The best Virgo modulus from five methods in Table 7 below (excluding the globular clusters!) gives  $(m - M) = 31.66 \pm 0.08$ . The mean turnover magnitude of the GCLF of eight  $E/S0$  Virgo cluster members gives  $\langle m_V^0 \rangle = 24.03 \pm 0.08$  from a compilation of Whitmore (1996). The resulting turnover magnitude of  $M_V^0 = -7.63 \pm 0.11$  is in fortuitous agreement with the RR-Lyr and Cepheid-based calibration of  $M_V^0 = -7.62 \pm 0.08$  (Sandage & Tammann 1995) from two *spiral* galaxies (Milky Way and M31). This is the first direct evidence that the mean turnover luminosity of the GCLF is the same in early-type galaxies and spirals to within the measuring accuracy. A point of considerable worry is on the other hand that Whitmore (1996) finds the turnover magnitude of seven *early-type* Fornax galaxies to be 0.23 mag *brighter* than that of eight certain Virgo cluster members, although at least the early-type Fornax members are certainly more distant (cf. Sect. 3).

The  $D_n - \sigma$  method, normally applied to ellipticals, was extended to the bulges of S0 and spiral galaxies by Dressler (1987). Using the bulges of the Galaxy, M31, and M81 as local calibrators, one obtains  $(m - M)_{\text{Virgo}} = 31.85 \pm 0.19$  (Tammann 1988).

Novae are potentially powerful distance indicators through their luminosity-decline rate relation. Using the Galactic calibration of Cohen (1985), Capaccioli et al. (1989) have found the apparent distance modulus of M31 to be  $(m - M)_{\text{AB}} = 24.58 \pm 0.20$  (i.e. somewhat less than indicated by Cepheids). From six novae in three Virgo ellipticals Pritchett & van den Bergh (1987) concluded that the cluster is more distant by  $7.0 \pm 0.4$  mag than the apparent modulus of M31, implying  $(m - M)_{\text{Virgo}} = 31.58 \pm 0.45$ . The result carries still low weight, but is interesting because it is based on novae exclusively. HST observations, although time-consuming, of novae in the Virgo cluster could much improve this *independent* result.

For some time it has been thought that the luminosity function of the shells of *planetary nebulae* (PNe) in the light of the  $\lambda 5007 \text{ \AA}$  emission line had a sharp universal cutoff at  $M_{5007} = -4.48$  mag. On the basis of this assumption a very small Virgo cluster modulus of  $(m - M) = 30.84$  was suggested (Jacoby, Ciardullo & Ford 1990). However, it was pointed out that the luminosity of the brightest PN shells depends on the population size, i.e. on the luminosity of the parent galaxy, in agreement with statistical expectations (Bottinelli et al. 1991). In fact it was shown that the available PN data agree perfectly well with a Virgo cluster modulus of  $(m - M) \sim 31.6$  if allowance is made for the population size effect (Tammann 1993). Numerically simulated luminosity functions of the shell luminosities have since confirmed that the peak luminosities depend on the sample size *and* on the population age (Méndez et al. 1993). It has therefore been proposed to use essentially the *shape* of the  $\lambda 5007 \text{ \AA}$  luminosity function (Soffner et al. 1996), but no results of this new method are available yet for the Virgo cluster.

Surface brightness fluctuations (SBF) of ellipticals and of the bulges of spirals have also been proposed as distance indicators (Tonry & Schneider 1988). The first “test” has remained rather unconvincing, spreading the *elliptical* Virgo cluster members over an interval of 12 to 24 Mpc (Tonry, Ajhar & Luppino 1990); this interval was interpreted as real although early-type galaxies are known to be concentrated in the cores of galaxy clusters. Moreover the individual distances correlate with the  $Mg_2$  index (Lorenz et al. 1993). A re-evaluation of the method has remedied these objections (Tonry et al. 1997), but

TABLE 7. The Virgo cluster modulus from various methods

method	$(m - M)_{\text{Virgo}}$	Hubble type
Cepheids	$31.47 \pm 0.21$	S
TF	$31.69 \pm 0.15$	S
SNe Ia	$31.62 \pm 0.16$	S
Globular Cl.	$31.67 \pm 0.15$	E
$D_n - \sigma$	$31.85 \pm 0.19$	E, S0
Novae	$31.46 \pm 0.40$	E
mean	$31.66 \pm 0.08$	

some of the results remain unconvincing. For instance the modulus distance of the Virgo cluster and the Leo group is suggested to be  $0.89 \pm 0.08$  mag as compared to  $1.25 \pm 0.13$  mag from Table 3. More telling is still the impossible SFB distance of the Virgo cluster of  $(m - M) = 31.03 \pm 0.05$  (!). This is not only  $0.66 \pm 0.09$  mag less than from all other evidence (cf. Table 7 below), but it would also imply a mean absolute magnitude of  $M_B(\text{max}) = -18.93 \pm 0.14$  mag of the eight standard type Ia SNe of the Virgo cluster, which is excluded by the seven Cepheid-calibrated SNe Ia giving  $M_B(\text{max}) = -19.53 \pm 0.07$  (Saha 1996) and by all existing type Ia models (cf. Section 4). The conclusion that the SFB distances beyond  $(m - M) \sim 30.0$  are too small by  $\geq 0.5$  mag will soon be decisively tested by NGC 7331, for which the SFB method gives a quite small distance of  $(m - M) = 30.39 \pm 0.10$  (Tonry et al. 1997), and for which a Cepheid distance will soon become available from HST (Hughes 1996).

### 2.6. The adopted Virgo cluster distance and the value of $H_0$ thereof

The valid evidence for the Virgo cluster distance is compiled in Table 7 from the preceding paragraphs. The six methods give very consistent results. This is remarkable in two respects. First, the methods include independent distance scales: the Cepheids, TF-distances, and SNe Ia depend on the zero point of the P-L relation of Cepheids, the  $D_n - \sigma$  method does not only depend on the Cepheid distances of M 31 and M 81, but also on the independent size of the Galactic bulge, the globular clusters rest on the P-L relation of RR Lyrae stars, and the novae rely on Cohen's (1985) Galactic calibration. Secondly the different distance determinations comprise spiral and E/S0 galaxies. This suggest that these two types of galaxies center about a common distance.

The best Virgo cluster modulus of  $(m - M)_{\text{Virgo}} = 31.66 \pm 0.08$  from Table 7 inserted into eq. (2.2) gives the large-scale value (out to at least  $\sim 10\,000 \text{ km s}^{-1}$ ) of

$$H_0 = 54 \pm 4. \quad (2.8)$$

### 3. $H_0$ from the Fornax or Coma clusters?

The Fornax cluster has in spite of being an irregular cluster a large fraction of early-type galaxies. The distribution in the sky of E/S0 and dE galaxies is shown in Fig. 6a, that of spiral and Im members in Fig. 6b. The relevant data are taken from Ferguson (1989). The well known Hubble type-density relation holds also here: The early-type galaxies are clearly more concentrated toward the cluster center; the spirals may still be more widely distributed as shown in Fig. 6b because the survey limits impose an

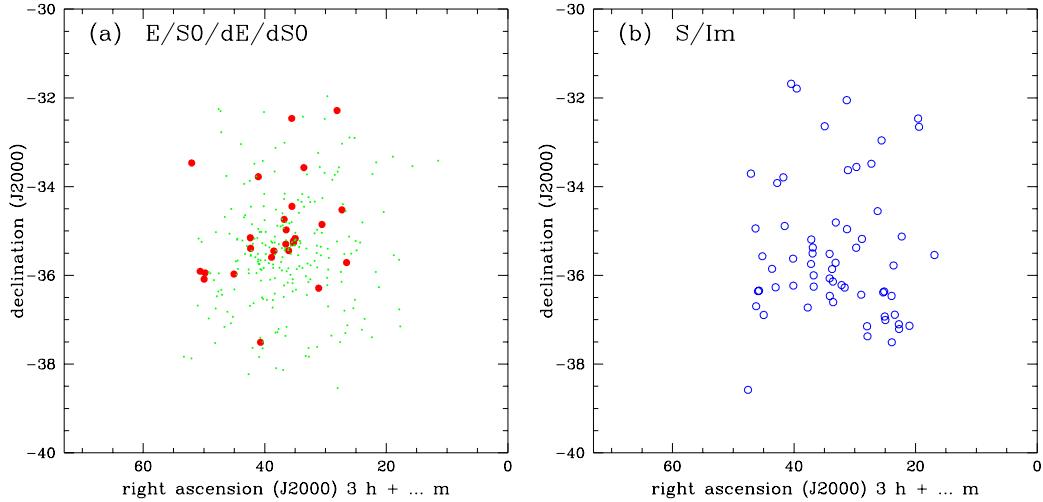


FIGURE 3. Apparent distribution of Fornax cluster galaxies in the sky: the E/S0/dE members are clearly concentrated towards the cluster center (a), whereas the spirals and irregulars are more widely distributed (b)

artificial cutoff. The cluster diameter traced by spirals is  $\sim 8^\circ$ , which corresponds – assuming sphericity – to a depth effect of 0.25 mag.

The distance to the Fornax cluster is notoriously poorly known. Even for the relative distance between Virgo and Fornax amazingly different values have been published. Such relative distances from the literature of the last 30 years are compiled in Table 8. The values are separated here for early-type galaxies and S/Im galaxies. The result is that the latter have smaller relative distance moduli than the E/S0/dE members by  $0.35 \pm 0.09$  mag. This is not due to a distance difference within the Virgo cluster, because here the two types of galaxies seem to be closely at the same mean distance. It would be premature to take the separation in space between the early-type and late-type Fornax members at face value, but it should be taken as a clear warning that the Fornax cluster may be elongated along the line of sight.

Velocities do not help to elaborate on the spatial structure of the Fornax cluster. The mean velocity of 41 E/S0/dE galaxies is  $v_{220} = 1323 \pm 48$  km s $^{-1}$  with a dispersion of  $\sigma = 307$  km s $^{-1}$ ; the mean velocity of 27 S/Im galaxies is  $v_{220} = 1436 \pm 66$  km s $^{-1}$  with an only slightly larger dispersion of  $\sigma = 343$  km s $^{-1}$ . The statistical agreement of the mean velocities can be interpreted as the early- and late-type members being at the same distance, but it could also be the result of the late-type members lying in the foreground and falling away towards the core of the cluster formed by early-type galaxies.

The mean velocity over all types is  $v_{LG} = 1366 \pm 50$  km s $^{-1}$  or  $v_{220} = 1338$  km s $^{-1}$ . The correction  $\Delta v_{220}$  for a self-consistent Virgocentric infall model is small because the Fornax cluster lies far away from the Virgo cluster at an angle of  $133^\circ$  from the latter. But that means also that the Fornax cluster may have its own peculiar motion of say  $\pm 350$  km s $^{-1}$  (see Section 2.1). – The cluster velocity in the CMB frame can be inferred from eq. (2.1) to be  $v^{\text{CMB}} = 1300 \pm 75$  km s $^{-1}$ , but this depends entirely on its distance relative to the Virgo cluster as given in Table 1. The reliability of the latter value may be questioned in view of the data in Table 8.

The conclusion from this is that the Fornax cluster is much less suited for the determination of  $H_0$  than the Virgo cluster. The possible spatial separation of the Fornax

TABLE 8. Relative distances between the Fornax and Virgo clusters

method	$\Delta(m - M)$ Fornax-Virgo spirals	$\Delta(m - M)$ Fornax-Virgo early types	reference
TF in $B$	$-0.40 \pm 0.10$		Schröder (1995)
TF in $I$	$-0.06 \pm 0.15$		Bureau et al. (1996)
SFB		$0.20 \pm 0.08$	Tonry et al. (1997)
globular clusters		$0.08 \pm 0.09$	Kohle et al. (1996)
surf. brightn.-lum. rel. of dEs		$0.40 \pm 0.12$	Jerjen (1995)
SN Ia		$0.36 \pm 0.12$	Sandage & Tammann (1995)
PNe		$0.24 \pm 0.10$	Mc Millan et al. (1993)
SN Ia		$0.09 \pm 0.14$	Hamuy et al. (1991)
SFB		$-0.16 \pm 0.13$	Tonry (1991)
globular clusters		$-0.11 \pm 0.20$	Bridges et al. (1991)
globular clusters		$-0.5 \pm 0.2$	Geisler & Forte (1990)
surf. brightn.-lum. rel. of dEs		$-0.19 \pm 0.15$	Bothun et al. (1989)
lum. vel. dispersion		$-0.48 \pm 0.58$	Pierce (1989)
lum. vel.-surf. brightn. rel.		$-0.14 \pm 0.43$	Pierce (1989)
TF in $H$	$-0.25 \pm 0.23$		Aaronson et al. (1989)
$D_n - \sigma$		$0.14 \pm 0.17$	Faber et al. (1989)
surf. brightn.-lum. rel. of E/dEs		$-0.02 \pm 0.20$	Ferguson & Sandage (1988a)
surf. brightn.-lum. rel. of dEs		$-0.5 \pm 0.2$	Caldwell & Bothun (1987)
$D_n - \sigma$ ( $Mg_2$ ) relation		$0.21 \pm (0.2)$	Dressler (1987)
TF in $H$	$-0.2 \pm 0.25$		Aaronson & Mould 1983
TF in $V, r, IV$	$-0.20 \pm 0.18$		Visvanathan (1983)
colour-lum. relation		$0.23 \pm 0.20$	Grierson (1982)
revision Visvanathan & Sandage (1977)		$0.16 \pm 0.17$	Aaronson et al. (1980)
globular clusters		$0.70 \pm 0.3$	de Vaucouleurs (1977)
colour-lum. relation		$0.32 \pm 0.23$	Visvanathan & Sandage (1977)
brightest cluster galaxies		$0.4 \pm 0.3$	Dawe & Dickens (1976)
brightest cluster galaxies		$0.36 \pm 0.25$	Sandage (1975)
Magn. & diam. of brightest gal.		$0.55 \pm 0.3$	de Vaucouleurs (1975)
brightest cluster galaxies		$0.62 \pm 0.25$	Sandage & Hardy (1973)
unweighted mean	$-0.22 \pm 0.05$	$0.13 \pm 0.07$	

members, its expected non-negligible peculiar motion, and the low weight of its  $v^{\text{CMB}}$  velocity call for great caution.

Three exercises to derive  $H_0$  from the Fornax cluster may illustrate the difficulties. Here we do not carry through the errors of  $H_0$  caused by the uncertain cluster velocity; the errors shown reflect only the errors introduced by the distance errors.

(1) A Cepheid distance of  $18.2 \pm 1.3$  Mpc has been published for NGC 1365 (Silbermann et al. 1996), an exceptionally large spiral galaxy in the Fornax field. If this value is confirmed and if it is taken – quite naively – as the distance of the cluster, one obtains with an adopted cluster velocity of  $v_{\text{LG}} = 1366 \text{ km s}^{-1}$   $H_0 = 75 \pm 6$ , or with the mean velocity of only the spirals ( $1436 \text{ km s}^{-1}$ )  $H_0 = 79 \pm 6$ .

(2) If one combines the modulus difference from Table 1 of  $(m - M)_{\text{Fornax}} - (m - M)_{\text{Virgo}} = 0.23 \pm 0.20$  with the best modulus of the Virgo cluster in Table 7, one finds  $(m - M)_{\text{Fornax}} = 31.89 \pm 0.22$ . This with  $v_{220} = 1338 \text{ km s}^{-1}$  gives  $H_0 = 56 \pm 6$ .

(3) A yet better distance can be determined from SN 1992A, a normal SN Ia in

TABLE 9. Data for the three SN Ia in Fornax cluster galaxies

SN (1)	galaxy (2)	$m_B(\text{max})$ (3)	$m_V(\text{max})$ (4)	$(m - M)$ (5)
1980N	NGC 1316	12.49	12.44	31.75
1981D	NGC 1316	12.59	12.40	31.78
1992A	NGC 1380	12.60	12.55	31.86
mean				31.80

NGC 1380. The cluster membership of this *early-type* (S0/a) galaxy, which lies close to the cluster core, cannot seriously be questioned. The relevant data for SN 1992A are given in Table 9 together with the data of SNe Ia 1980N and 1981D, which both occurred in the outlying Sa galaxy NGC 1316. The cluster membership of this galaxy is not certain, but the data for all *three* SNe Ia are so similar, that the inclusion of NGC 1316 only decreases, if anything, the inferred cluster distance.

The apparent magnitudes of the three SNe Ia in columns (3) and (4) in Table 9 are taken from Hamuy et al. (1996). The distance moduli in column (5) are the mean of the moduli in  $B$  and  $V$ . They are based on assumed absolute maximum magnitudes of  $M_B(\text{max}) = -19.31$  and  $M_V(\text{max}) = -19.26$ . These absolute magnitudes are 0.20 mag fainter than the mean of seven SNe Ia calibrated with Cepheids (Saha et al. 1997). The fainter luminosities are indicated here because the calibrating SNe Ia lie predominantly in late-type spirals and since SNe Ia in early-type galaxies are generally somewhat fainter (Suntzeff 1996). The resulting mean Fornax cluster distance of  $(m - M) = 31.80 \pm 0.20$  leads together with  $v^{\text{CMB}} = 1300 \text{ km s}^{-1}$  to  $H_0 = 57 \pm 6$ .

The conclusion of point (1) to (3) is that the main body of the Fornax cluster formed by early-type galaxies may well comply with  $H_0 \approx 55$ . If the small Cepheid distance of the large spiral NGC 1365 is correct, it lies certainly in the foreground, possibly together with many other spirals of the field as suggested by the data in Table 8. In any case the Fornax cluster, whose velocity in the CMB frame is in addition quite uncertain, cannot provide at present a meaningful value of  $H_0$ .

It is sometimes suggested to use the Coma cluster to derive  $H_0$  by adding the relative distance modulus  $\Delta(m - M)_{\text{Coma-Virgo}}$  to the Virgo cluster modulus  $(m - M)_{\text{Virgo}}$  and by assuming – without justification – that the peculiar motion of the Coma cluster is negligible. In view of the many cluster distances relative to the Virgo cluster and of their Hubble diagram in Fig. 1 this is now a step backwards. However, there is one independent distance information for the Coma cluster. From HST observations of the globular clusters of NGC 4881 Baum et al. (1995) found a minimum distance of  $r_{\text{Coma}} > 108 \pm 11 \text{ Mpc}$ . Since the depth effect of the cluster becomes vanishingly small at the distance of Coma, this result holds for the whole cluster. With  $\Delta(m - M)_{\text{Coma-Virgo}} = 3.80$  from Table 1 and from eq. (2.1) one finds a cosmic velocity of  $v_{\text{Coma}}^{\text{CMB}} = 6730 \text{ km s}^{-1}$  which gives, when combined with Baum's et al. (1995) lower distance limit,  $H_0 < 62 \pm 7$ . (Note that a smaller value of  $\Delta(m - M)_{\text{Coma-Virgo}}$ , as preferred by some authors, gives a lower value of  $v_{\text{Coma}}^{\text{CMB}}$  and hence of  $H_0$ .

TABLE 10.  $H_0$  determinations from field galaxies corrected for Malmquist bias

method	$H_0$	source
Tully-Fisher, distance-limited, ( $v_{LG} \leq 500 \text{ km s}^{-1}$ )	$50 \pm 5$	Richter & Huchtmeier (1984); Sandage (1988a, 1994)
Tully-Fisher, flux-limited (distant)	$< 60$	Sandage (1994)
M 101 look-alike diameters	$43 \pm 11$	Sandage (1993a)
M 31 look-alike diameters	$45 \pm 12$	Sandage (1993b)
luminosity classes of spirals	$56 \pm 5$	Sandage (1996a)
M 31, M 101 look-alike luminosity	$55 \pm 5$	Sandage (1996a)
Tully-Fisher (using magn.+diam.)	$55 \pm 5$	Theureau et al. (1996)
weighted mean	$53 \pm 3$	

#### 4. $H_0$ from field galaxies

Catalogues of field galaxies are flux-limited. This has the consequence, in the presence of intrinsic luminosity scatter, that the mean luminosity of the galaxies increases with distance. This does not only hold for galaxies in general, but also for galaxies of a fixed rotation velocity, velocity dispersion, colour etc. – wherever the dispersion is non-negligible. The unfailing signature of Malmquist bias is – if uncorrected – that it always leads to values of  $H_0$  increasing with distance, contrary to overwhelming evidence (for a discussion cf. Sandage 1995). Uncorrected distances of field galaxies (de Vaucouleurs 1979; Aaronson et al. 1986), which were said to require high values of  $H_0$ , lead indeed to an unrealistic increase of  $H_0$  with distance (Tammann 1987).

A simple-minded compensation for the Malmquist bias has been to take *distant* galaxies from a catalogue with a fainter flux limit than that of nearer ones (Sandage & Tammann 1975). The next step was to introduce a correction term to the mean absolute magnitudes which increases linearly with distance in the sense to make distant galaxies brighter (Sandage, Tammann & Yahil 1979). Sophisticated Malmquist corrections to TF distances were applied by Bottinelli et al. (1986), Lynden-Bell et al. (1988), Sandage (1988a), Federspiel, Sandage & Tammann (1994), Giovanelli (1996b), Theureau et al. (1996) and others. In a case study Hendry & Simmons (1990) have considered 184 Sc galaxies of luminosity class I and derived a “naive” value of  $H_0 = 86$  (using an arbitrary zeropoint) if  $\sigma_M$  were zero; however, allowing for an internal dispersion of  $\sigma_M = 0.5$  and 1.0 mag they found a maximum likelihood solution of  $H_0 = 56$  and  $H_0 = 44$ , respectively.

A representative sample of Malmquist-corrected  $H_0$  determinations from field galaxies with typically  $v < 4000 \text{ km s}^{-1}$  is compiled in Table 10.

It remains to point to an unexpected paradox. The 15 or so galaxies outside the Local Group with known Cepheid distances give, in combination with their velocities  $v_{LG}$  or  $v_{220}$ , a *very local* value of  $H_0 \approx 70 \pm 5$  (cf. Giovanelli 1996b), i.e.  $H_0$  seems to be larger within the Virgo cluster circle than outside. This is contrary to all previous (mainly Malmquist-generated) claims of  $H_0$  increasing outwards, but also contrary to physical expectations in view of the local overdensity. Judgement about this result must await a still larger sample of Cepheid distances. The result is questioned by the TF distances of a *complete* sample of spiral galaxies with  $v_{LG} < 500 \text{ km s}^{-1}$  (Kraan-Korteweg & Tammann 1979) giving  $H_0 = 50 \pm 5$  (cf. Table 10).

## 5. $H_0$ from geometric and physical methods

The accurate geometric distance determination of LMC via the ring of SN 1987A (Panagia et al. 1996) is very important for the extragalactic distance scale because it supports the Cepheid distance of LMC of  $(m - M)^0 = 18.50$  to within 0.1 mag.

Other geometric distances come from VLBI observations of the radio remnants of SNe II. They have provided two distances so far, viz. of the nearby galaxy M 81 (Marcaide et al. 1995) and the Virgo galaxy NGC 4321 (M 100; Bartel 1991). The accuracy, however, does not match that of the Cepheids.

There are plans to measure proper motions in external galaxies by means of masers (e.g. Elitzur 1992), but much progress is still needed to derive distances which are directly useful for  $H_0$ . The velocity drift of water maser lines close to the very massive nucleus of NGC 4258 are attributed to the centripetal acceleration of gravity; this together with VLBI observations provides a distance of this galaxy of  $6.4 \pm 0.9$  Mpc (Miyoshi et al. 1995). – Sparks (1994) has proposed to determine the locus of maximum polarization of the scattered light echos of SNe; HST observations for this project are under way.

Important results are coming from theoretical models of SNe. From expanding-photosphere models of SNe II Schmidt et al. (1994) have argued that  $H_0 = 73 \pm 6$ , but their crucial dilution factor (i.e. deviation from black-body radiation) has been criticized by Baron et al. (1995; 1996) who argue for  $H_0 < 50$ . There is less controversy for SNe Ia. Höflich et al. (1996) have derived spectrum-fitting expanding-atmosphere luminosities for five SNe Ia for which Cepheid distances are available. Their mean absolute magnitudes of  $M_B(\text{max}) = -19.40 \pm 0.20$  and  $M_V(\text{max}) = -19.37 \pm 0.18$  are fainter by only  $\Delta M_B = 0.05 \pm 0.21$  and  $\Delta M_V = 0.10 \pm 0.19$  than Saha's (1996) Cepheid-calibrated mean luminosities, the only significant difference being that Höflich's et al. (1996) luminosities have larger scatter than actually observed. The five theoretical models combined with the Hubble diagram of SNe Ia beyond  $v = 1000 \text{ km s}^{-1}$  gives  $H_0 = 60 \pm 6$ . Branch et al. (1996) find an expanding-photosphere of  $M_B(\text{max}) = -19.49 \pm 0.35$  and  $M_V(\text{max}) = -19.42 \pm 0.35$  for SN Ia 1981B and from its  $^{56}\text{Ni}$  mass the independent value  $M_B(\text{max}) \approx M_V(\text{max}) \approx -19.4 \pm 0.2$ . This should be compared with the marginally fainter calibration from Cepheids (Saha 1996, Table 1). Ruiz-Lapuente (1996) has derived absolute magnitudes from the late nebular spectra of two SNe Ia (SN 1989B and SN 1990N) which are marginally fainter by  $0.26 \pm 0.25$  mag than their Cepheid-calibrated luminosities.

The Sunyaev-Zeldovich effect (SZ) provides distances of X-ray clusters the quality of which depends on a detailed gas model and a number of assumptions, one of which being the sphericity of the gas volume. Several distance determinations of the cluster A 2218 yield  $H_0 = 45 \pm 20$  (McHardy et al. 1990; Birkinshaw & Hughes 1994; Jones 1994; Lasenby & Hancock 1995). The unweighted mean of five other clusters from various authors (listed by Raphaeli 1995) and of the Coma cluster (Herbig, Lawrence & Readhead 1995) is  $H_0 = 60 \pm 15$ . Lasenby (1996) derives from two clusters, A 2218 and A 1413,  $H_0 = 42^{+12}_{-9}$ . Raphaeli (1995) concludes in his review that the method will be become competitive through a compilation of high-sensitivity X-ray and SZ data on a large cluster sample.

Distances of gravitationally lensed double quasars can also be determined if the time delay between the variability of the two images and the mass distribution of the deflector lens are known (Refsdal 1964). These distant objects do not yield, however, the present value of  $H$ . One has therefore to assume some value of  $q_0$  to obtain  $H_0$ , but the uncertainty is only of the order of  $\sim 10\%$ . The double quasar QSO 0957+561 with a now well-determined time delay of  $\tau = 420$  days (Pelt et al. 1996; Thomson & Schild 1997)

TABLE 11. Different determinations of  $H_0$ 

method	range km s <sup>-1</sup>	$H_0$ km s <sup>-1</sup> Mpc <sup>-1</sup>	external error
field galaxies	4000	53 ± 3	±10
Virgo cluster	10000	54 ± 4	±8
SNeIa	30000	58 ± 4	±6
physical methods	large	50-65	(±15)
adopted		55	±7

gives values of  $H_0 < 70$  (Dahle, Maddox, & Lilje 1994; Turner 1996); the most recent analysis gives  $H_0 = 63 \pm 12$  at the 95% confidence level (Kundić et al. 1996). Further progress is expected from narrow double quasars like B0218+357 (Patniak, Porcas & Browne 1995) because the deflecting mass model is here better constraint. Present data give for the least mentioned object  $\tau = 12 \pm 3$  days (Corbett et al. 1995) and  $H_0 \approx 60$  (Nair 1995).

The measurement of the “Doppler peaks” in the CMB fluctuation spectrum opens a new dimension to the meaning of  $H_0$ . A comparison of the inferred and observed values of  $H_0$  will have a profound effect on our understanding of the very early universe. The first results, giving  $30 < H_0 < 50$  (or at most 70), are most encouraging (Lasenby 1996).

## 6. Conclusions

It is now possible to determine  $H_0$  along three lines of attack as described in Sections 2, 4, and 5. The results are repeated in Table 11. Also shown is the high-weight determination of  $H_0$  from SNeIa (Saha 1996).

The agreement of the different determinations is so good that the external error of  $H_0 = 55 \pm 7$  (±13%) seems to be secure unless one postulates a conspiracy of all data.

It should be noted, however, that  $H_0$  from field galaxies and SNeIa depends entirely on the calibration via Cepheids and that Cepheids carry also much of the weight of the multiple distance determinations of the Virgo cluster leading directly to a large-scale value of  $H_0$ .

Yet Cepheids are the best and least controversial distance indicators. The zeropoint of their period-luminosity (P-L) relation has moved by only 0.07 mag over an interval of 30 years (Kraft 1961; Sandage & Tammann 1968; Feast & Walker 1987; Madore & Freedman 1991). The calibration, using only Galactic Cepheids, gives a distance modulus of LMC of  $(m - M) = 18.50 \pm 0.10$  (Sandage & Tammann 1968; Feast & Walker 1987), which is independently confirmed to better than 0.10 mag through the shell of the SN 1987A in LMC (Panagia et al. 1996), RR Lyr stars [(Walker (1993) with the calibration of Sandage (1993c)], the red-giant tip (Lee, Freedman, & Madore 1993), the Baade-Becker-Wesselink method in *BVIJHK* for RR Lyr stars (Laney & Stobie 1992), and the *I* magnitude diameters of Cepheids (di Benedetto 1995).

The slope of the P-L relation taken from LMC is uncritical as long as the available Cepheids cover a sufficient period interval. Metallicity effects on the P-L relation are small (Freedman & Madore 1990; Chiosi, Wood, & Capitanio 1993; Sandage 1996b); any such effects enter with low weight because no strongly metal-deficient galaxies are

considered here. Selection bias (Sandage 1988b) can be avoided if the Cepheids span a sufficient period interval.

These or similar arguments have led to a general consent that Cepheids, as far as they can reliably be observed, are the most fundamental distance indicators known. The zero point error of the extragalactic distance scale introduced by the Cepheid P-L relation is hence certainly less than 10% (0.2 mag). The concordant evidence from stepping up the distance scale through the Virgo cluster, field galaxies, and SN Ia to the large scale value of  $H_0$  makes it unlikely that the final error could be larger than 13%.

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